Doublet final focus

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Abstract. A doublet scheme is designed in a self consistent analytical way for the low β insertion of a μ^+-μ^- collider. It assumes focusing strengths of same magnitude and opposite signs in the quadrupoles. At the matching point, the β values are equal and the α values opposite. Two solutions using superconducting or permanent quadrupoles are discussed.

INTRODUCTION

The high luminosity of a muon collider requires a beta value as low as 3 mm at the interaction point (1). Several optical modules can be contemplated to achieve a round beam in the interaction region. The one discussed here is a doublet made of identical quadrupoles of opposite focusing strengths. Their thin lens parameters obey a very simple conjugation law and the generalization to finite length elements results from the solution of a transcendental equation. In order to reduce the chromaticity of the final focus, the quadrupoles have to be as close as possible to the interaction point yet satisfy the detector constraints. Permanent and superconducting quadrupoles are compared and the parameters of a superconducting scheme are given.

THIN LENS DOUBLET

The final focus of a round beam is a doublet made of two quadrupoles Q_1 and Q_2 of opposite focal lengths separated by a distance d. The theory of the doublet shows that the horizontal and vertical β-functions can be transported from the interaction point where they are both equal to β^* to a matching point where the canonical conditions

$$\beta_h - \beta_v = 0$$
$$\alpha_h + \alpha_v = 0$$

are fulfilled if the distances from the interaction point to the next quadrupole Q_1 and from the matching point to the other quadrupole Q_2 are equal to the same value l^* and if the conjugation relation (2)

$$f = \sqrt{dl^*}$$

is satisfied. The β-value at the matching point is then the same as the value at Q_2 in the absence of doublet

$$\beta = \frac{(d + l^*)^2}{\beta^*}$$

and the absolute value of α

$$\alpha = \frac{d + l^*}{\beta^*}\sqrt{\frac{d}{l^*}}$$

is reduced with respect to the α–value at Q_2 by the factor $\sqrt{d / l^*}$ which is called k and plays a basic role in the discussion. All those expressions assume that β^* is small.

THICK LENS DOUBLET

The distance between quadrupoles is limited by the length l of the quadrupoles and its minimum value is precisely l. It will then be assumed that

$$d = l = k^2 l^*$$

and the focal length is re-written

$$f = kl^*$$

When the doublet is made of thick lenses, the global properties are maintained but the parameter is no longer a focal length but a phase u, solution of the transcendental equation

$$u(\cosh u \sin u - \cos u \sinh u) = a \cos u \cosh u \,,$$

where a is the ratio between the length of the quadrupole and the length of the final drift space

$$a = \frac{l}{l_0}\,,$$

a is related to k through the relation

$$a = \frac{2k^2}{2 - k^2}\,,$$

and the constant k must be smaller than $\sqrt{2}$. The numerical solution of the transcendental equation for various values of a shows that the fitting function

$$u_0 = 1.11205\sqrt{k} - 0.0108984\,k + 0.0905792\,k^3$$

is a very precise analytical solution in the interval (0, $\sqrt{2}$). The numerical solutions together with the fitting function are represented in Figure 1.

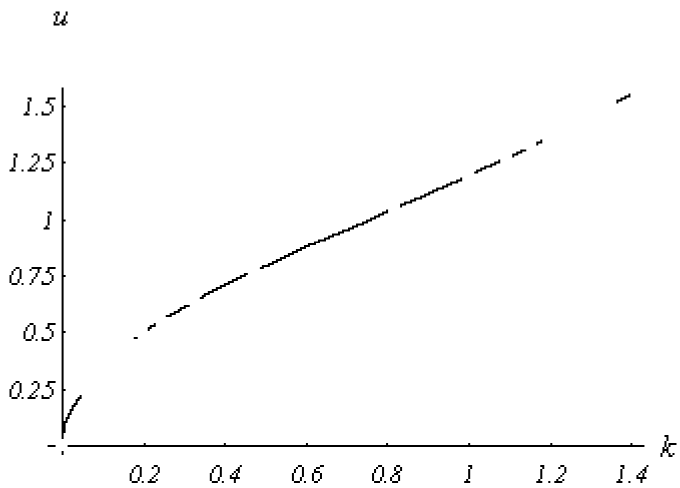


FIGURE 1. Variation of the phase *u* with the ratio *k*.

The phase u is related to the focusing strength K_L of a long quadrupole and to the length l through the relation

$$u = \sqrt{K_L}\,l$$

whereas the focal length of a thin lens depends on K_S and l according to the formula

$$f = \frac{1}{K_S l}$$

All the calculations of K in the thin lens approximation must then be corrected by the factor u_0^2/k to be valid for real quadrupoles. The focusing strength K is proportional to the gradient G of the magnetic field

$$K = \frac{e}{p}G$$

e and p being the charge and the momentum of the particle. The gradient can be expressed as the ratio B/r where B is the magnetic field at a beam radius r of sufficiently large emittance.

$$r = \sqrt{\frac{\varepsilon_N \beta}{\gamma}},$$

ε_N is the normalized emittance and γ the Lorentz factor. The β-function takes its maximum value β_2 at Q_2 and is equal to

$$\beta_2 = \frac{l^{*2}}{\beta^*}(1 + k + k^2)^2$$

By performing all the substitutions and equating the optical and magnetic expressions of the focal length, the final length is found to be

$$l^* = \frac{u_0{}^2(1 + k + k^2)}{k^4 B} \frac{p}{e} \sqrt{\frac{\varepsilon_N}{\gamma \beta^*}}$$

The parameter k must be such that the contribution of Q_2 to the chromaticity

$$\frac{\beta_2}{4\pi f}$$

is as low as possible. It turns out that the function of k which enters the chromaticity is

$$\frac{u_0{}^2(1 + k + k^2)^2}{k^5}$$

and it decreases with k. The quadrupoles must thus be as close as possible to the interaction point. The distance l_0 from the IP is given by the blind angle θ allowed by the detector:

$$l_0 = \frac{qr}{\theta}$$

where q is the ratio of the outer radius of the quadrupole to the beam radius. The previous relation can then be written as an equation in k:

$$1 - \frac{k^2}{2} = \frac{q}{\theta} \sqrt{\frac{\varepsilon_N}{\gamma \beta^*}} \left(1 + k + k^2\right)$$

DOUBLET PARAMETERS

The determination of the parameters starts with the evaluation of k from the above equation, then the lengths l^* and l are calculated and the properties of the doublet in the thin lens approximation are found with a dedicated function of the

BeamOptics (3) program. Last, the real beam envelope in thick quadrupoles is derived by scaling the focusing strengths of the thin lens model.

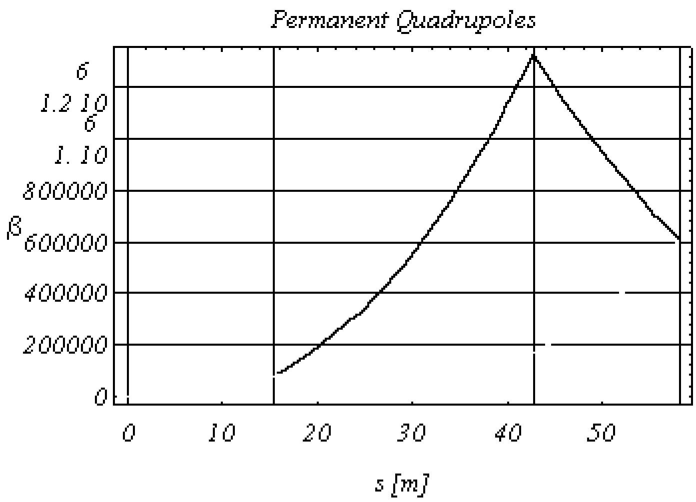
The permanent and superconducting solutions will first be compared in the thin lens approximation for the general parameters

$$\varepsilon_N = 5\ 10^{-5}\ \mathrm{m}\quad p = 2\ \mathrm{TeV}\quad \beta^* = 3\ \mathrm{mm}\quad \theta = .1\ \mathrm{radian}$$

The normalized emittance corresponds to the r.m.s. value estimated after cooling. This is too small for a realistic design but too large emittances would lead to optic parameters very hard, if not impossible, to achieve at least for a doublet and for so small a β^* value. For permanent and superconducting quadrupoles, the field at the edge of the beam and the factor *q* are chosen as listed in the following table

	B [T]	q
permanent quadrupoles	1.2	3
superconducting quadrupoles	4	20

and the pattern of the β-functions is shown in Figure 2. Although the permanent magnets are closer to the interaction point, their relatively low field forces them to be much longer than the superconducting quadrupoles and the ratio of the peak β-values is almost 10. The chromaticity of the permanent magnet scheme is also higher in absolute value than in the super-conducting case: 5000 against 1500. It has nevertheless to be checked that a field of 4 Tesla is possible. As a comparison, the field in LHC low-β quadrupoles (4) is only 1.35 T at a radius equal to 10 times the beam r.m.s. value.



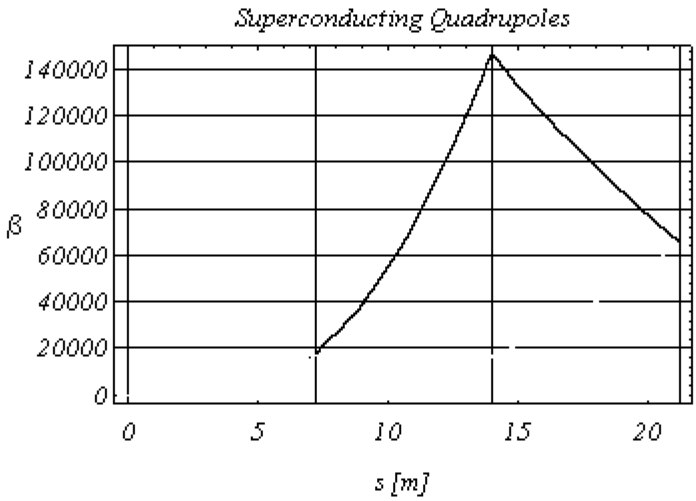


Figure 2. β-variations in two types of doublet final focus in the thin lens approximation.

The parameters of the super-conducting magnets are retained for the thick lens doublet (Figure 3). It turns out that the matching conditions are fulfilled but that the β-values are 10% higher about and that the doublet acceptance is reduced accordingly. Another aspect of this scheme is that the length of the matching straight section, which is equal to the interaction region length, is much longer than for permanent magnets and there is thus room to put other quadrupoles to complete the matching with the regular part of the lattice. The variations of the beam size are plotted in Figure 4, the outer radius of the quadrupole tank is slightly inferior to 400 mm and fits the solid angle of .1 radian since the interaction length is 3.83 m.

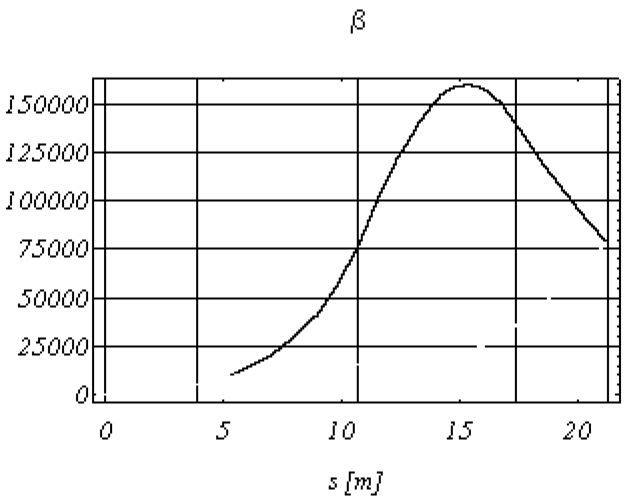


Figure 3. β-variations in a doublet made of thick super-conducting quadrupoles.

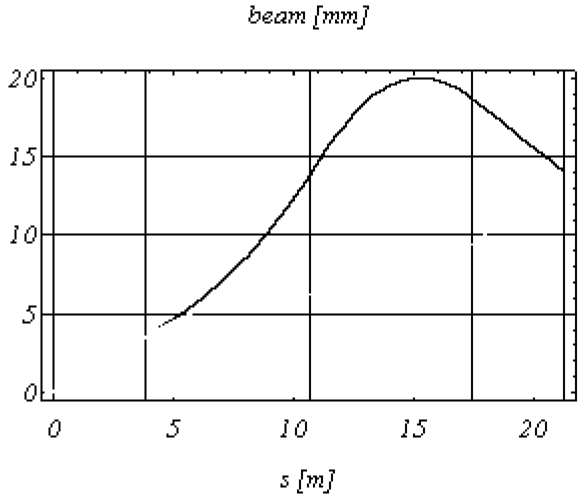


Figure 4. Beam size in a doublet made of thick super-conducting quadrupoles.

CONCLUSION

An analytical technique has been presented to derive the parameters of a doublet final focus. Its application to a muon collider shows that a doublet scheme is questionable in the present status of the parameters especially because the beam size has been taken to its r.m.s. value to maintain the maximum β-value within the already scary hundred kilometer range. Some improvement might be gained in reducing the aperture of the quadrupole next to the interaction point to shorten its length yet maintaining its focusing strength. If the doublet has the advantage to permit the optimum bunch length, it is severely limited by the defocusing effect in one plane. A triplet suffers from the same restriction. In contrast, a single lens avoids the dramatic de-focusing at the cost of a flat beam; the net consequence is either a loss of luminosity or a shorter bunch length. The machine performance however is not only determined by such basic machine parameters as β^*, it is also affected by its dynamic aperture and the quadrupole technology. It is thus necessary to carefully compare several schemes and, among them, the singlet scheme, to get a realistic global design.

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